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Abduction, Deduction, and Induction:

Their implications to quantitative methods

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Abstract

While quantitative methods have been widely applied by social scientists such as sociologists, psychologists, and economists, their philosophical premises and assumptions are rarely examined. The philosophical ideas introduced by Charles Sanders Peirce (1839-1914) are helpful for researchers in understanding the application of quantitative methods specific to the foundational concepts of deduction, abduction and induction. In the Peircean logical system the nature of knowledge and reality relate to each of these concepts: the logic of abduction and deduction contribute to our conceptual understanding of a phenomenon, while the logic of induction adds quantitative details to our conceptual knowledge. At the stage of abduction, the goal is to explore data, find a pattern, and suggest a plausible hypothesis; deduction is to refine the hypothesis based upon other plausible premises; and induction is empirical substantiation. This article seeks to investigate the premises, limitations and applications of deduction, abduction and induction within quantitative methodology.

Fisher (1935, 1955) considered significance testing as “inductive inference” and argued that this approach is the source of all knowledge. On the other hand, Neyman (1928, 1933a, 1933b) maintained that only deductive inference was appropriate in statistics as shown in his school of hypothesis testing tradition. However, both deductive and inductive methods have been criticized for various limitations such as their tendency to explain away details that should be better understood and their incapability of generating new knowledge (Hempel, 1965; Josephson & Josephson, 1994; Thagard & Shelley, 1997). In the view of the Peircean logical system, one may say the logic of abduction and deduction contribute to our conceptual understanding of a phenomena (Hausman, 1993), while the logic of induction provides empirical support to conceptual knowledge. In other words, abduction, deduction, and induction work together to explore, refine and substantiate research questions.

Although abduction is central in the Peircean logical system, Peirce by no means downplayed the role of deduction and induction in inquiry. Peirce had studied the history of philosophy thoroughly and was influenced by a multitude of schools of logic (Hoffmann, 1997). Peirce explained these three logical processes (1934/1960) as, “Deduction proves something must be. Induction shows that something actually is operative; abduction merely suggests that something may be” (Vol. 5, p.171). Put another way: Abduction plays the role of generating new ideas or hypotheses; deduction functions as evaluating the hypotheses; and induction is justifying the hypothesis with empirical data (Staat, 1993).

This article attempts to apply abduction, which was introduced by Peirce a century ago, to offer a more comprehensive logical system of research methodology. Therefore, we will evaluate the strengths and weaknesses of the preceding three logical processes under Peircean direction, and point to implications for the use of exploratory data analysis (EDA) and quantitative research within this philosophical paradigm.

It is important to note that the focus of this article is to extend and apply Peircean ideas into research methodologies in an epistemological fashion, not to analyze the original meanings of Peircean ideas in the manner of historical study. Almder (1980) contended that Peirce wrote in a style that could lead to confusion. Not surprisingly, many scholars could not agree on whether Peircean philosophy is a coherent system or a collection of disconnected thoughts (Anderson, 1987). In response to Weiss (1940) who charged some philosophers with distorting and dismembering the Peircean philosophy, Buchler (1940) contended that cumulative growth of philosophy results from the partial or limited acceptance of a given philosopher's work through discriminating selection. One obvious example of extending the Pericean school is the "inference to the best explanation" (IBE) proposed by Harman (1965) based upon the Peircean idea of abduction. While the "classical" abduction is considered a logic of discovery, IBE is viewed as a logic of justification (Lipton, 1991). But in the context of debating realism and anti-realism, de Regt (1994) criticized that Peircean philosophy was mis-used to the extent that the "inference to the *best* explanation" had become the "inference to the *only* explanation." This article is concerned with neither history of philosophy nor discernment of various interpretations of the Peircean system; rather I adopted the position suggested by Buchler, and thus Peircean ideas on abduction, deduction, and induction are discussed through discriminating selection.

Abduction

Premises of abduction

Before discussing the logic of abduction and its application, it is important to point out its premises. In the first half of the 20th century, verificationism derived from positivism dominated the scientific community. For positivists unverifiable beliefs should be rejected. However, according to Peirce, researchers must start from somewhere, even though the starting point is an unproven or unverifiable assumption. This starting point of scientific consciousness is private fancy a flash of thought, or a wild hypothesis. But it is the seed of creativity (Wright, 1999). This approach is very different from positivism and opens more opportunities for inquirers (Callaway, 1999). In the essay The Fixation of Belief, (1877) Peirce said that we are satisfied with stable beliefs rather than doubts. Although knowledge is fallible in nature, and in our limited lifetime we cannot discover the ultimate truth, we will still fix our beliefs at certain points. At the same time, Peirce did not encourage us to relax our mind and not pursue further inquiry. Instead, he saw seeking knowledge as interplay between doubts and beliefs, though he did not explicitly use the Hegelian term "dialectic."

The logic of abduction

Grounded in the fixation of beliefs, the function of abduction is to look for a pattern in a surprising phenomenon and to suggest a plausible hypothesis. The following example illustrates the function of abduction:

The surprising phenomenon, B, is observed.

But if A were true, B would be a matter of course.

Hence there is a reason to suspect that A might be true.

By the standard of deductive logic, the preceding reasoning is clearly unacceptable for it is contradicted with a basic rule of inference in deduction, namely, *Modus Poenes*. Following this rule, the legitimate form of reasoning takes the route as follows:

A is observed.

If A, then B.

Hence, B is accepted.

Modus Ponens is commonly applied in the context of conducting a series of deduction for complicated scientific problems. For example, A; (A → B); B; (B → C); C; (C → D); D…etc. However, Peirce started from the other end:

B is observed.

If A, then B.

Hence, A can be accepted.

Logicians following deductive reasoning call this *the fallacy of affirming the consequent*. Consider this example. It is logical to assert that "It rains; if it rains, the floor is wet; hence, the floor is wet." But any reasonable person can see the problem in making statements like: "The floor is wet; if it rains, the floor is wet; hence, it rains." Nevertheless, in Peirce's logical framework this abductive form of argument is entirely valid, especially when the research goal is to discover plausible explanations for further inquiry (de Regt, 1994). In order to make inferences to the best explanation, the researcher must need a set of plausible explanations, and thus, abduction is usually formulated in the following mode:

The surprising phenomenon, X, is observed.

Among hypotheses A, B, and C, A is capable of explaining X.

Hence, there is a reason to pursue A.

At first glance, abduction is an educated guess among existing hypotheses. Thagard and Shelley (1999) clarified this misconception. They explained that unifying conceptions were an important part of abduction, and it would be unfortunate if our understanding of abduction were limited to more mundane cases where hypotheses are simply assembled. Abduction does not occur in the context of a fixed language, since the formation of new hypotheses often goes hand in hand with the development of new theoretical terms such as “quark,” and “gene.” Indeed, Peirce (1934/1960) emphasized that abduction is the only logical operation that introduces new ideas.

Some philosophers of science such as Popper (1968) and Hempel (1966) suggested that there is no logic of discovery because discovery relies on creative imagination. Hempel used Kekule’s discovery of the hexagonal ring as an example. The chemist Kekule failed to devise a structural formula for the benzene molecule in spite of many trials. One evening he found the solution to the problem while watching the dance of fire in his fireplace. Gazing into the flames, he seemed to see atoms dancing in snakelike arrays and suddenly related this to the molecular structure of benzene. This is how the hexagonal ring was discovered. However, it is doubtful whether this story supports the notion that there is no logic of discovery. Why didn’t other people make a scientific breakthrough by observing the fireplace? Does the background knowledge that had been accumulated by Kekule throughout his professional career play a more important role to the discovery of the hexagonal ring than a brief moment in front of a fireplace? The dance of fire may serve as an analogy to the molecular structure that Kekule had contemplated. Without the deep knowledge of chemistry, it is unlikely that anyone could draw inspiration by the dance of fire.

For Peirce, progress in science depends on the observation of the right facts by minds furnished with appropriate ideas (Tursman, 1987). Definitely, the intuitive judgment made by an intellectual is different from that made by a high school student. Peirce cited several examples of remarkable correct guesses. All success is not simply luck. Instead, the opportunity was taken by the people who were prepared:

a). Bacon's guess that heat was a mode of motion;

b). Young's guess that the primary colors were violet, green and red;

c). Dalton's guess that there were chemical atoms before the invention of microscope (cited in Tursman, 1987).

By the same token to continue the last example, the cosmological view that "atom" is the fundamental element of the universe, introduced by ancient philosophers Leucippus and Democritus, revived by Epicurus, and confirmed by modern physicists, did not result from a lucky guess. Besides the atomist theory, there were numerous other cosmological views such as the Milesian school, which proposed that the basic elements were water, air, fire, earth … etc. Atomists were familiar with them and provided answers to existing questions based on the existing framework (Trundle, 1994).

Peirce stated that classification plays a major role in making a hypothesis, that is the characters of phenomenon are placed into certain categories (Peirce, 1878b). Although Peirce is not a Kantian (Feibleman 1945), Peirce endorsed Kant's categories in Critique of Pure Reason (Kant, 1781/1969) to help us to make judgments of the phenomenal world. According to Kant, human thought and enlightenment are dependent on a limited number of a priori perceptual forms and ideational categories, such as causality, quality, time and space. Also, Peirce agreed with Kant that things have internal structure of meaning. Abductive activities are not empirical hypotheses based on our sensory experience, but rather the very structure of the meanings themselves (Rosenthal, 1993). Based on the Kantian framework, Peirce (1867/1960) later developed his "New list of categories." For Peirce all cognition, ranging from perception to logical reasoning, is mediated by “elements of generality.” (Peirce, 1934/1960). Based upon the notion of categorizing general elements, Hoffman (1997) viewed abduction as a search for a mode of perception while facing surprising facts.

Applications of abduction

Abduction can be well applied to quantitative research, especially Exploratory Data Analysis (EDA) and Exploratory statistics (ES), such as factor rotation in Exploratory Factor Analysis and path searching in Structural Equation Modeling (Glymour, Scheines, Spirtes, & Kelly, 1987; Glymour & Cooper, 1999). Josephson and Josephson (1994) argued that the whole notion of a controlled experiment is covertly based on the logic of abduction. In a controlled experiment, the researchers control alternate explanations and test the condition generated from the most plausible hypothesis. However, abduction shares more common ground with EDA than with controlled experiments. In EDA, after observing some surprising facts, we exploit them and check the predicted values against the observed values and residuals (Behrens, 1997). Although there may be more than one convincing pattern, we "abduct" only those that are more plausible for subsequent controlled experimentation. Since experimentation is hypothesis-driven and EDA is data-driven, the logic behind them are quite different. The abductive reasoning of EDA goes from data to hypotheses while inductive reasoning of experimentation goes from hypothesis to expected data. By the same token, in Exploratory Factor Analysis and Structural Equation Modeling, there might be more than one possible way to achieve a fit between the data and the model; again, the researcher must “abduct” a plausible set of variables and paths for modeling building.

Shank (1991), Josephson and Josephson (1994), and Ottens and Shank (1995) related abductive reasoning to detective work. Detectives collect related “facts” about people and circumstances. These facts are actually shrewd guesses or hypotheses based on their keen powers of observation. In this vein, the logic of abduction is in line with EDA. In fact, Tukey (1977, 1980) often related EDA to detective work. In EDA, the role of the researcher is to explore the data in as many ways as possible until a plausible "story" of the data emerges. EDA is <u>not</u> “fishing” significant results from all possible angles during research: it is not trying out everything.

Rescher (1978) interpreted abduction as an opposition to Popper's falsification (1963). There are millions of possible explanations to a phenomenon. Due to the economy of research, we cannot afford to falsify every possibility. As mentioned before, we don't have to know everything to know something. By the same token, we don't have to screen every false thing to dig out the authentic one. During the process of abduction, the researcher should be guided by the elements of generality to extract a proper mode of perception.

Summary

In short, abduction can be interpreted as conjecturing the world with appropriate categories, which arise from the internal structure of meanings. The implications of abduction for researchers as practiced in EDA and ES, is that the use of EDA and ES is neither exhausting all possibilities nor making hasty decisions. Researchers must be well equipped with proper categories in order to sort out the invariant features and patterns of phenomena. Quantitative research, in this sense, is not number crunching, but a thoughtful way of dissecting data.

Deduction

Premise of deduction

Aristotle is credited as the inventor of deduction (Trundle, 1994). Deduction presupposes the existence of truth and falsity. Quine (1982) stated that the mission of logic is the pursuit of truth, which is the endeavor to sort out the true statements from the false statements. Hoffmann (1997) further elaborated this point by saying that the task of deductive logic is to define the validity of one truth as it leads to another truth. It is important to note that the meaning of truth in this context does not refer to the ontological, ultimate reality. Peirce made a distinction between truth and reality: Truth is the understanding of reality through a self-corrective inquiry process by the whole intellectual community across time. On the other hand, the existence of reality is independent of human inquiry (Wiener, 1969). In terms of ontology, there is one reality. In regard to methodology and epistemology, there is more than one approach and one source of knowledge. Reality is "what is" while truth is "what would be." Deduction is possible because even without relating to reality, propositions can be judged as true or false within a logical and conceptual system.

Logic of deduction

Deduction involves drawing logical consequences from premises. An inference is endorsed as deductionaly valid when the truth of all premises guarantees the truth of conclusion. For instance,

First premise: All the beans from the bag are white (True).

Second premise: These beans are from this bag (True).

Conclusion: Therefore, these beans are white (True). (Peirce, 1986).

According to Peirce, deduction is a form of analytic inference and of this sort are all mathematical demonstrations (1986).

Limitations of deduction

There are several limitations of deductive logic. First, deductive logic confines the conclusion to a dichotomous answer (True/False). A typical example is the rejection or failure of rejection of the null hypothesis. This narrowness of thinking is not endorsed by the Peircean philosophical system, which emphasizes the search for a deeper insight of a surprising fact.

Second, this kind of reasoning cannot lead to the discovery of knowledge that is not already embedded in the premise (Peirce, 1934/1960). In some cases the premise may even be tautological--true by definition. Brown (1963) illustrated this weakness by using an example in economics: An entrepreneur seeks maximization of profits. The maximum profits will be gained when marginal revenue equals marginal cost. An entrepreneur will operate his business at the equilibrium between marginal cost and marginal revenue.

The above deduction simply tells you that a rational man would like to make more money. There is a similar example in cognitive psychology:

Human behaviors are rational.

One of several options is more efficient in achieving the goal.

A rational human will take the option that directs him to achieve his goal (Anderson, 1990).

The above two deductive inferences simply provide examples that a rational man will do rational things. The specific rational behaviors have been included in the bigger set of generic rational behaviors. Since deduction facilitates analysis based upon existing knowledge rather than generating new knowledge, Josephson and Josephson (1994) viewed deduction as truth preserving and abduction as truth producing.

Third, deduction is incomplete as we cannot logically prove all the premises are true. Russell and Whitehead (1910) attempted to develop a self-sufficient logical-mathematical system. In their view, not only can mathematics be reduced to logic, but also logic is the foundation of mathematics. However, Gödel (1947/1986) showed that we cannot even establish all mathematics by deductive proof. To be specific, it is impossible to have a self-sufficient system as Russell and Whitehead postulated. Any lower order theorem or premise needs a higher order theorem or premise for substantiation; and no system can be complete and consistent at the same time. Deduction alone is clearly incapable of establishing the empirical knowledge we seek.

Peirce reviewed Russell's book "Principles of Mathematics" in 1903, but he only wrote a short paragraph with vague comments. Nonetheless, based on Peirce's other writings on logic and mathematics, Haack (1993) concluded that Peirce would be opposed to Russell and Whitehead's notion that the epistemological foundations of mathematics lie in logic. It is questionable whether the logic or the mathematics can fully justify deductive knowledge. No matter how logical a hypothesis is, it is only sufficient within the system; it is still tentative and requires further investigation with external proof.

This line of thought posed a serious challenge to researchers who are confident in the logical structure of statistics. Mathematical logic relies on many unproven premises and assumptions. Statistical conclusions are considered true only given that all premises and assumptions that are applied are true. In recent years many Monte Carlo simulations have been conducted to determine how robust certain tests are, and which statistics should be favored. The reference and criteria of all these studies are within logical-mathematical systems without any worldly concerns. For instance, the Fisher protected t-test is considered inferior to the Ryan test and the Tukey test because it cannot control the inflated Type I error very well (Toothaker, 1993), not because any psychologists or educators made a terribly wrong decision based upon the Fisher protected t-test. Pillai-Bartlett statistic is considered superior to Wilk's Lambda and Hotelling-Lawley Trace because of much greater robustness against unequal covariance matrices (Olson, 1976), not because any significant scientific breakthroughs are made with the use of Pillai-Bartlett statistic. For Peirce this kind of self-referential deduction cannot lead to progress in knowledge. Knowing is an activity which is by definition involvement with the real world (Burrell, 1968).

As a matter of fact, the inventor of deductive syllogisms, Aristotle, did not isolate formal logic from external reality and he repeatedly admitted the importance of induction. It is not merely that the conclusion is deduced correctly according to the formal laws of logic. Aristotle assumes that the conclusion is verified in reality. Also, he devoted attention to the question: How do we know the first premises from which deduction must start? (Copleston, 1946/85; Russell, 1945/72)

Certain development of quantitative research methodology is not restricted by logic. Actually, statistics is by no means pure mathematics without interactions with the real world. Gauss discovered the Gaussian distribution through astronomical observations. Fisher built his theories from applications of biometrics and agriculture. Survival analysis or the hazard model is the fruit of medical and sociological research. Differential item functioning (DIF) was developed to address the issue of reducing test bias.

Last but not least, for several decades philosophers of science have been debating about the issue of under-determination, a problematic situation in which several rival theories are empirically equivalent but logically incompatible (de Regt, 1994; Psillos, 1999). Under-determination is no stranger to quantitative researchers, who constantly face model equivalency in factor analysis and structural equation modeling. Under-determination, according to Leplin (1997), is a problem rooted in the limitations of the hypothetico-deductive methodology, which is disconfirmatory in nature. For instance, the widely adopted hypothesis testing is based on the logic of computing the probability of obtaining the observed data (D) given that the theory or the hypothesis (H) is true (P(D|H)). At most this mode of inquiry can inform us when to reject a theory, but not when to accept one. Thus, quantitative researchers usually draw a conclusion using the language in this fashion: “Reject the hypothesis” or “fail to reject the hypothesis,” but not “accept the hypothesis” or “fail to accept the hypothesis.” Passing a test is not confirmatory if the test is one that even a false theory would be expected to pass. At first glance it may be strange to say that a false theory could lead to passing of a test, but that is how under-determination occurs. Whenever a theory is proposed for predicting or explaining a phenomenon, it has a deductive structure. What is deduced may be an empirical regularity that holds only statistically, and thus, the answer by deduction works well for the true theory as for the false ones.

Summary

For Peirce, deduction alone is a necessary condition, but not a sufficient condition of knowledge. Peirce (1934/1960) warned that deduction is applicable only to the ideal state of things. In other words, deduction alone can be applied to a well-defined problem, but not an ill-defined problem, which is more likely to be encountered by researchers. Nevertheless, deduction performs the function of clarifying the relation of logical implications. When well-defined categories result from abduction, premises can be generated for deductive reasoning.

Induction

Premise of induction

For Peirce, induction is founded on the premise that inquiry is a self-corrective inquiry process by the whole intellectual community across time. Peirce stressed the collective nature of inquiry by saying “No mind can take one step without the aid of other minds” (1934/1960, p.398). Unlike Kuhn's (1962) emphasis on paradigm shift and incommensurability between different paradigms, Peirce stressed the continuity of knowledge. First, knowledge does not emerge out of pure logic. Instead, it is a historical and social product. Second, Peirce disregarded the Cartesian skepticism of doubting everything (DesCartes, 1641/1964). To some extent we have to fix our beliefs on those positions that are widely accepted by the intellectual community (Peirce, 1877).

Kuhn proposed that the advancement of human knowledge is a revolutionary process in which new frameworks overthrow outdated frameworks. Peirce, in contrast, considered knowledge to be continuous and cumulative. Rescher (1978) used the geographical-exploration model as a metaphor to illustrate Peirce's idea: The replacement of a flat-world view with a global-world view is a change in conceptual understanding, or a paradigm shift. After we have discovered all the continents and oceans, measuring the height of Mount Everest and the depth of the Nile river is adding details to our conceptual knowledge. Although Kuhn's theory looks glamorous, as a matter of fact, paradigm shifts might occur only once in several centuries. The majority of scholars are just adding details to existing frameworks. Knowledge is self-corrective insofar as we inherit the findings from previous scholars and refine them.

Logic of induction

Induction introduced by Francis Bacon is a direct revolt against deduction. Bacon (1620/1960) found that people who use deductive reasoning rely on the authority of antiquity (premises made by masters), and the tendency of the mind to construct knowledge within the mind itself. Bacon criticized deductive users as spiders for they make a web of knowledge out of their own substance. Although the meaning of deductive knowledge is entirely self-referent, deductive users tend to take those propositions as assertions. Propositions and assertions are not the same level of knowledge. For Peirce, abduction and deduction only gives propositions, however, self-correcting induction provides empirical support to assertions.

Inductive logic is often based upon the notion that probability is the relative frequency in long run and a general law can be concluded based on numerous cases. For examples,

A1, A2, A3 ... A100 are B.

A1, A2, A3 ... A100 are C.

Therefore, B is C.

Or

A1, A2, A3, … A100 are B.

Hence, all A are B.

Nonetheless, the above is by no mean the only way of understanding induction. Induction could also take the form of prediction:

A1,A2,A3…A100 are B.

Thus, A101 will be B.

Limitations of induction

Hume (1777/1912) argued that things are inconclusive by induction because in infinity there are always new cases and new evidence. Induction can be justified, if and only if, instances of which we have no experience resemble those of which we have experience. Thus, the problem of induction is also known as “the skeptical problem about the future” (Hacking, 1975). Take the previous argument as an example. If A101 is not B, the statement "B is C" will be refuted.

We never know when a regression line will turn flat, go down, or go up. Even inductive reasoning using numerous accurate data and high power computing can go wrong, because predictions are made only under certain specified conditions (Samuelson, 1967). For instance, based on the case studies in the 19th century, sociologist Max Weber (1904/1976) argued that capitalism could be developed in Europe because of the Protestant work ethic; other cultures like the Chinese Confucianism are by essence incompatible with capitalism. However, after World War Two, the emergence of Asian economic powers such as Taiwan, South Korea, Hong Kong and Singapore disconfirmed the Weberian hypothesis.

Take the modern economy as another example. Due to American economic problems in the early '80s, quite a few reputable economists made gloomy predictions about the U.S. economy such as the takeover of American economic and technological throne by Japan. By the end of the decade, Roberts (1989) concluded that those economists were wrong; contrary to those forecasts, in the 80’s the U.S. enjoyed the longest economic expansion in its history. In the 1990s, the economic positions of the two nations changed: Japan experienced recession while America experienced expansion.

“The skeptical problem about the future” is also known as “the old riddle of induction.” In a similar vein to the old riddle, Goodman (1954/1983) introduced the “new riddle of induction,” in which conceptualization of kinds plays an important role. Goodman demonstrated that whenever we reach a conclusion based upon inductive reasoning, we could use the same rules of inference, but different criteria of classification, to draw an opposite conclusion. Goodman’s example is: We could conclude that all emeralds are green given that 1000 observed emeralds are green. But what would happen if we re-classify “green” objects as “blue” and “blue” as “green” in the year 2020? We can say that something is “grue” if it was considered “green” before 2020 and it would be treated as “blue” after 2020. We can also say that something is “bleen” if it was counted as a “blue” object before 2020 and it would be regarded as “green” after 2020. Thus, the new riddle is also known as “the grue problem.”

In addition, Hacking (1999) cited the example of “child abuse,” a construct that has been taken for granted by many Americans, to demonstrate the new riddle. Hacking pointed out that actually the concept of “child abuse” in the current form did not exist in other cultures. Cruelty to children just emerged as a social issue during the Victorian period, but “child abuse” as a social science concept was formulated in America around 1960. To this extent, Victorians viewed cruelty to children as a matter of poor people harming their children, but to Americans child abuse was a classless phenomenon. When the construct “child abuse” became more and more popular, many American adults recollected childhood trauma during psychotherapy sessions, but authenticity of these child abuse cases was highly questionable. Hacking proposed that “child abuse” is a typical example of how re-conceptualization in the future alters our evaluations of the past.

Another main theme of the new riddle focuses on the problem of projectibility. Whether an "observed pattern" is projectible depends on how we conceptualize the pattern. Skyrms (1975) used a mathematical example to illustrate this problem: If this series of digits (1, 2, 3, 4, 5) is shown, what is the next projected number? Without any doubt, for most people the intuitive answer is simply "6." Skyrms argued that this seemingly straight-forward numeric sequence could be populated by this generating function: (A-1)(A-2)(A-3)(A-4)(A-5)+A. Let's step through this example using an Excel spreadsheet. In Cell A1 to A10 of the Excel spreadsheet, enter 1-10, respectively. Next, in Cell B1 enter the function "=(A1-1)*(A1-2)*(A1-3)*(A1-4)*(A1-5)+A1" and this function will yield "1." Afterwards, select Cell B1 and "drag" the cursor downwards to Cell B10; it will copy the same function to B2, B3, B4…B10. As a result, (B1 to B5) will correspond to (A1 to A10), which are (1, 2, 3, 4, 5). However, the sixth number in Column B, which is 126, substantively deviates from the intuitive projection. All numbers in the cells below B6 are also surprising. Skyrms pointed out that whatever number we want to predict for the sixth number of the series, there is always a generating function that can fit the given members of the sequence and that will yield the projection we want. This indeterminacy of projection is a mathematical fact.

Figure 1. Generating function for a sequence of numbers

B1 fx =(A1-1)*(A1-2)*(A1-3)*(A1-4)*(A1-5)+A1

	A	B	C	D	E	F
1	1	1				
2	2	2				
3	3	3				
4	4	4				
5	5	5				
6	6	126				
7	7	727				
8	8	2528				
9	9	6729				
10	10	15130				

Furthermore, the new riddle, which is considered an instantiation of the general problem of under-determination in epistemology, is germane to quantitative researchers in the context of “model equivalency” and “factor indeterminacy.” (DeVito, 1997; Forster, 1999; Forster & Sober, 1994; Kieseppa, 2001; Muliak, 1996; Turney, 1999). Specifically, the new riddle and other philosophical notions of under-determination illustrate that all scientific theories are under-determined by the limited evidence in the sense that the same phenomenon can be equally well-explained by rival models that are logically incompatible. In factor analysis, for example, whether adopting a one-factor or a two-factor model may have tremendous impact on subsequent inferences. In curving-fitting problem, whether using the Akaike’s Information Criterion or the Bayesian Information Criterion is crucial in the sense that these two criteria could lead to different conclusions. Hence, the preceding problem of model selection criteria in quantitative-based research is analogous to the problem of re-conceptualization of “child abuse” and the problem of projectibility based upon generating functions. At the present time, there are no commonly agreed solutions to either the new riddle or the model selection criteria.

Second, induction suggests the possible outcome in relation to events in long run. This is not definable for an individual event. To make a judgment for a single event based on probability like "your chance to survive this surgery is 75 percent" is nonsense. In actuality, the patient will either live or die. In a single event, not only the probability is indefinable, but also the explanatory power is absent. Induction yields a general statement that explains the event of observing, but not the facts observed. Josephson and Josephson (1994) gave this example: “Suppose I choose a ball at random (arbitrarily) from a large hat containing colored balls. The ball I choose is red. Does the fact that all of the balls in the hat are red explain why this particular ball is red? No…’All A’s are

B's' cannot explain why 'this A is a B' because it does not say anything about how its being an A is connected with its being a B." (p.20)

As mentioned before, induction also suggests probability as a relative frequency. In the discussion of probability of induction, Peirce (1986) raised his skepticism to this idea: "The relative probability…is something which we should have a right to talk about if universes were as plenty as blackberries, if we could put a quantity of them in a bag, shake them well up, draw out a sample, and examine them to see what proportion of them had one arragement and what proportion another." (pp. 300-301). Peirce is not alone in this matter. To many quantitative researchers, other types of interpretations of probability, such as the subjective interpretation and the propensity interpretation, should be considered.

Third, Carnap, as an inductive logician, knew the limitation of induction. Carnap (1952) argued that induction might lead to the generalization of empirical laws but not theoretical laws. For instance, even if we observe thousands of stones, trees and flowers, we never reach a point at which we observe a molecule. After we heat many iron bars, we can conclude the empirical fact that metals will bend when they are heated. But we will never discover the physics of expansion coefficients in this way.

Indeed, superficial empirical-based induction could lead to wrong conclusions. For example, by repeated observations, it seems that heavy bodies (e.g. metal, stone) fall faster than lighter bodies (paper, feather). This Aristotelian belief had misled European scientists for over a thousand years. Galileo argued that indeed both heavy and light objects fall at the same speed. There is a popular myth that Galileo conducted an experiment in the Tower of Pisa to prove his point. Probably he never performed this experiment. Actually this experiment was performed by one of Galileo's critics and the result supported Aristotle's notion. Galileo did not get the law from observation, but by a chain of logical arguments (Kuhn, 1985). Again, superficial induction runs the risk of getting superficial and incorrect conclusion.

Quantitative researchers have been warned that high correlations among variables may not be meaningful. For example, if one plots GNP, educational level, or anything against time, one may see some significant but meaningless correlation (Yule, 1926). As Peirce (1934/1960) pointed out, induction cannot furnish us with new ideas because observations or sensory data only lead us to superficial conclusions but not the "bottom of things" (p.878).

Last but not least, induction as the sole source of reliable knowledge was never inductively concluded. An Eighteenth century British moral philosopher Thomas Reid embraced the conviction that the Baconian philosophy or the inductive method could be extended from the realm of natural science to mind, society, and morality. He firmly believed that through an inductive analysis of the faculties and powers by which the mind knows, feels, and wills, moral philosophers could eventually establish the scientific foundations for morality. However, some form of circularity was inevitable in his argument when induction was validated by induction. Reid and his associates counter-measured this challenge by arguing that the human mental structure was designed explicitly and solely for an inductive means of inquiry (cited in Bozeman, 1977). However, today the issue of inductive circularity remains unsettled because psychologists still could not reach a consent pertaining to the human reasoning process. While some psychologists found that the frequency approach appears to be more natural to learners in the context of quantitative reasoning (Gigerenzer, 2003; Hoffrage, Gigerenzer, & Martignon, 2002), some other psychologists revealed that humans have conducted inquiry in the form of Bayesian network by the age of five (Gopnik & Schulz, 2004). Proclaiming a particular reasoning mode as *the* human mind structure in a hegemonic tone, needless to say, would lead to immediate protest.

Summary

For Peirce induction still has validity. Contrary to Hume's notion that our perception of events is devoid of generality, Peirce argued that the existence we perceive must share generality with other things in existence. Peirce's metaphysical system resolves the problem of induction by asserting that the data from our perception are not reducible to discrete, logically and ontologically independent events (Sullivan, 1991). In addition, for Peirce all empirical reasoning is essentially making inferences from a sample to a population; the conclusion is merely approximately true (O'Neill, 1993). Forster (1993) justified this view with the Law of Large Numbers. On one hand, we don't know the real probability due to our finite existence. However, given a large number of cases, we can approximate the actual probability. We don't have to know everything to know something. Also, we don't have to know every case to get an approximation. This approximation is sufficient to fix our beliefs and lead us to further inquiry.

Conclusion

In summary, abduction, deduction and induction have different merits and shortcomings. Yet the combination of all three reasoning approaches provides researchers a powerful tool of inquiry. For Peirce a reasoner should apply abduction, deduction and induction altogether in order to achieve a comprehensive inquiry. Abduction and deduction are the conceptual understanding of phenomena, and induction is the quantitative verification. At the stage of abduction, the goal is to explore the data, find out a pattern, and suggest a plausible hypothesis with the use of proper categories; deduction is to build a logical and testable hypothesis based upon other plausible premises; and induction is the approximation towards the truth in order to fix our beliefs for further inquiry. In short, abduction creates, deduction explicates, and induction verifies.

A good example of their application can be found in the use of the Bayesian Inference Network (BIN) in psychometrics (Mislevy, 1994). According to Mislevy, the BIN builds around deductive reasoning to support subsequent inductive reasoning from realized data to probabilities of states. Yet abductive reasoning is vital to the process in two aspects. First, abductive reasoning suggests the framework for inductive reasoning. Second, while the BIN is a tool for reasoning deductively and inductively within the posited structure, abduction is required to reason about the structure. Another example can be found in the mixed methodology developed by Johnson and Onwuegbuzie (2004). Research employing mixed methods (quantitative and qualitative methods) makes use of all three modes of reasoning. To be specific, its logic of inquiry includes the use of induction in pattern recognition, which is commonly used in thematic analysis in qualitative methods, the use of deduction, which is concerned with quantitative testing of theories and hypotheses, and abduction, which is about inferences to the best explanation based on a set of available alternate explanations. It is important to note that researchers do not have to follow a specific order in using abduction, deduction, and induction. In Johnson and Onwuegbuzie's framework, abduction is a tool of justifying the results at the end rather than generating a hypothesis at the beginning of a study.

One of the goals of this chapter is to illustrate a tight integration among different modes of inquiry, and its implication to exploratory and confirmatory analyzes. Consider this counter-example. Glymour (2001) viewed widespread applications of factor analysis as a sign of system-wide failure in social sciences in terms of causal interpretations. As a strong advocate of structural equation modeling, which is an extension of confirmatory factor analysis, Glymour is very critical of this exploratory factor modeling approach. By reviewing the history of psychometrics, Glymour stated that reliability (a stable factor structure) was never a goal of early psychometricians. Thurstone faced the problem that there were many competing factor models that were statistically equivalent. In order to “saving the phenomena” (to uniquely determining the factor loadings), he developed the criterion of the simple structure, which has no special measure-theoretic virtue or special stability properties. In addition, on finite samples, factor analysis may fail to recover the true causal structure because of statistical or algorithmic artifacts. On the contrary, Glymour (2005) developed path-searching algorithms for model building, in which huge data sets are collected; automated methods are employed to search for regularities in the data; hypotheses are generated and tested as they go along. The last point is especially important because for Gkymour there is no sharp distinction between the exploratory and confirmatory steps.

However, even if path-searching algorithms are capable of conducting hypothesis generation and testing altogether, it is doubtful whether the process is totally confirmatory and nothing exploratory. In a strict sense, even CFA is a mixture of exploratory and confirmatory techniques, in which the end product is derived in part from theory and in part from a re-specification based on the analysis of the fitness indices. The same argument is well-applied to path-searching. According to Peirce, in the long run scientific inquiry is a self-correcting process; earlier theories will inevitably be revised or rejected by later theories. In this sense, all causal conclusions, no matter how confirmatory they are, must be exploratory in nature because these confirmed conclusions are subject to further investigations. In short, it is the author’s belief that integration among abduction, induction, and deduction, as well as between exploratory and confirmatory analyses, could enable researchers to conduct a thorough investigation.

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